TRIANGLE AND ITS PROPERTIES

Learning Objectives:

At the end of this lesson, students will be able to:

- Identify the median and altitude of a triangle.
- Establish Exterior Angle Property of a triangle.
- Solve problems based on Exterior Angle Property.
- Establish Angle Sum Property of a triangle.
- Solve problems based on Angle Sum Property.
- Explore the properties of equilateral and isosceles triangles.
- Check whether a triangle exists by exploring the relation between the lengths of the sides of the triangle.
- State Pythagoras Property for right-angled triangles.
- Identify a right-angled triangle by verifying the Pythagoras Property.

Topics Covered:

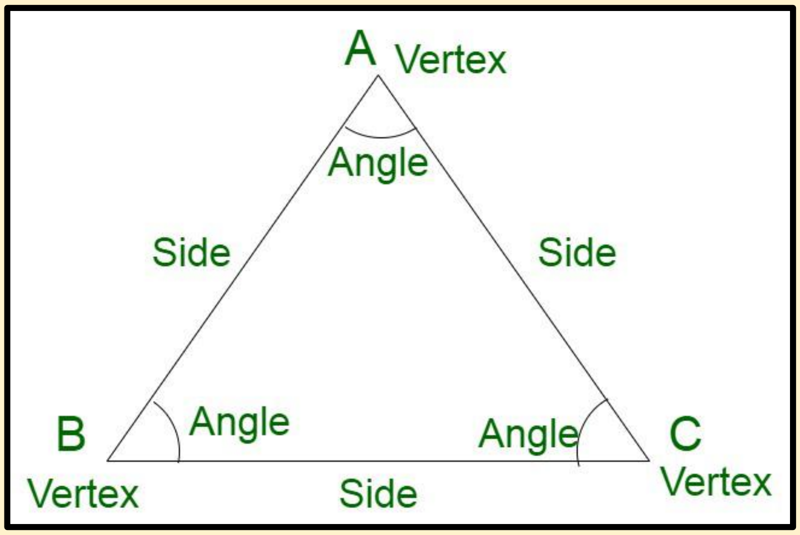
- ❖ Median of a Triangle
- ❖ Altitude of a Triangle
- ❖ Exterior Angle of a Triangle and its Property
- ❖ Angle Sum Property of a Triangle
- ❖ Two Special Triangles : Equilateral and Isosceles
- ❖ Sum of the Lengths of two Sides of a Triangle
- ❖ Right-angled Triangles and Pythagoras Property

Warm – up:

We already know that a triangle is a simple closed figure made of three line segments.

A triangle has:

- three sides
- three angles
- three vertices



In the given Δ ABC,

Sides: AB, BC and CA

Angles: ∠A, ∠B and ∠C

Vertices: A, B and C

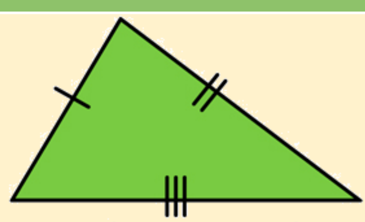
Triangles can be classified on the basis of their sides and angles.

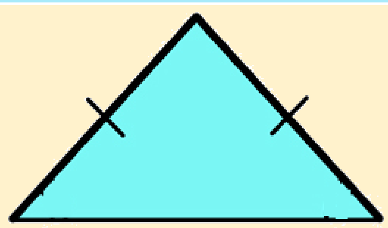
On the basis of sides

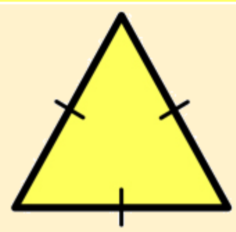
Scalene Triangle:
A triangle in which all the sides are different in length is called a scalene triangle.

Isosceles Triangle
A triangle in which two sides are equal in length is called an isosceles triangle.

Equilateral Triangle
A triangle in which all the sides are equal in length is called an equilateral triangle.





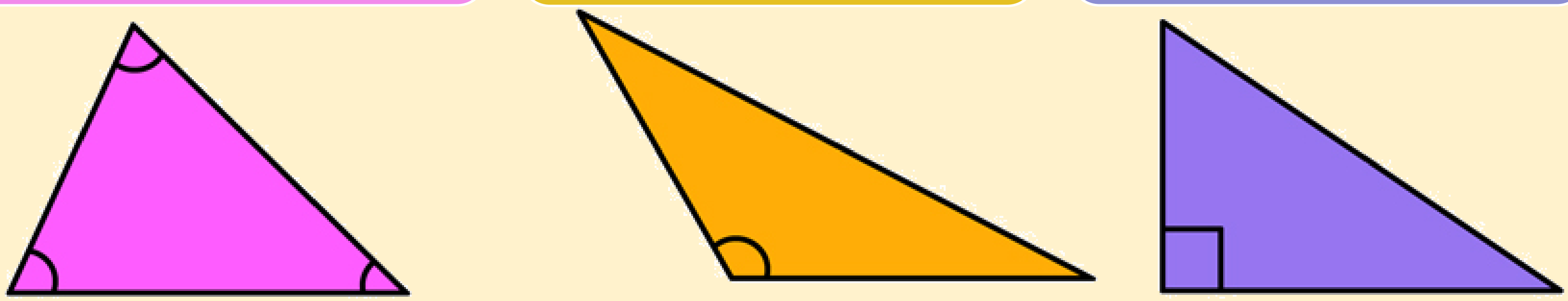


On the basis of angles

Acute-angled Triangle:
A triangle in which all the angles are acute is called an acute-angled triangle.

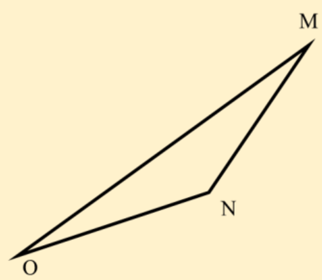
Obtuse-angled Triangle
A triangle in which one of the angles is obtuse is called an obtuse-angled triangle.

Equilateral Triangle
A triangle in which all the angles are equal in measure is called an equilateral triangle.



From Theory to Practice:

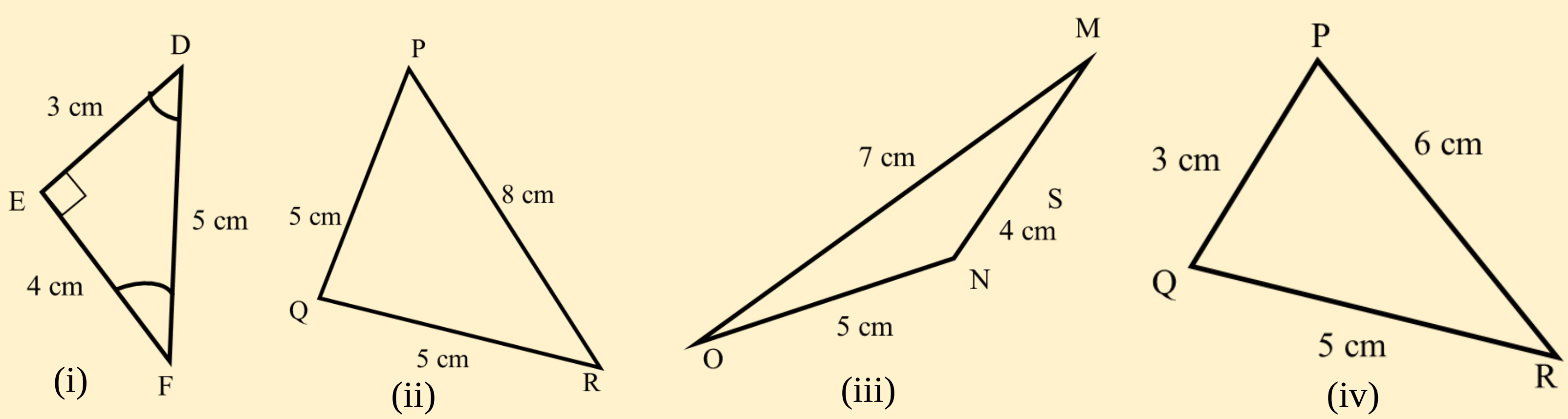
1. Find out the side opposite the ∠N, the angle made by the side MO and NO, and vertex opposite to MN.



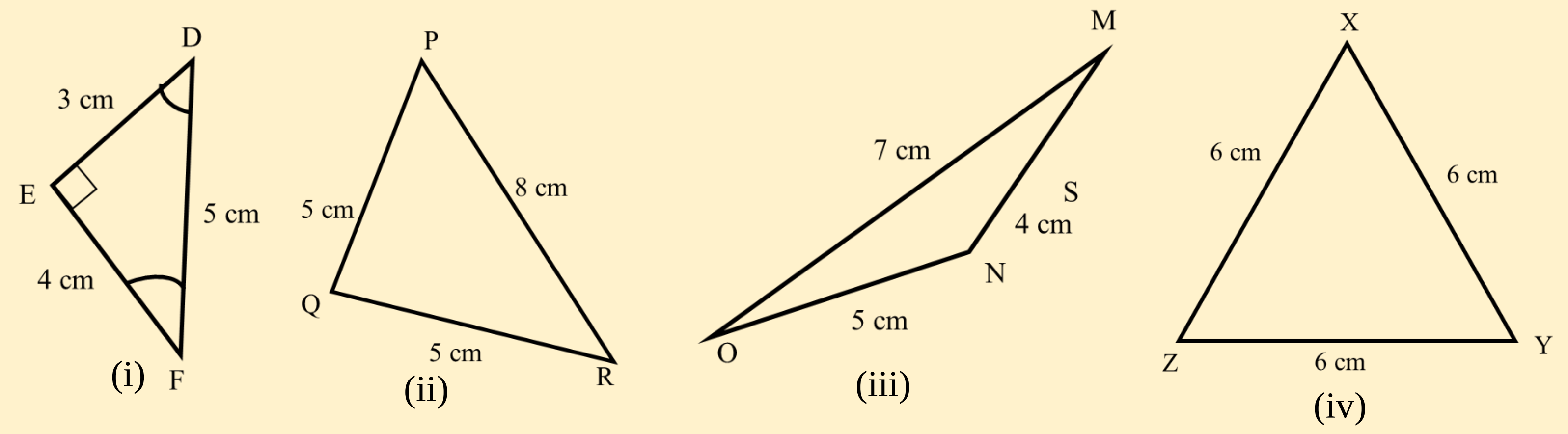
Solution:

The side opposite the ∠N is MO.
The angle between the sides MO and NO is ∠O.
The vertex opposite to MN is O.

2. Classify each of the following triangles on the basis of sides and angles.



Solution:



The given triangles can be classified on the basis of sides and angles as follows.

On the basis of sides:
Δ DEF is a scalene triangle as the lengths of all its sides are unequal.
DE ≠ EF ≠ DF

Δ PQR is an isosceles triangle as the lengths of its two sides are equal.
PQ = QR ≠ PR

Δ MNO is a scalene triangle as the lengths of all its sides are unequal.
MN ≠ NO ≠ MO

Δ XYZ is an equilateral triangle as the lengths of all its sides are equal.
XY = YZ = XZ

On the basis of angles:
Δ DEF is a right-angled triangle as one of its angles is right angle, i.e., 90°.
∠E = 90°

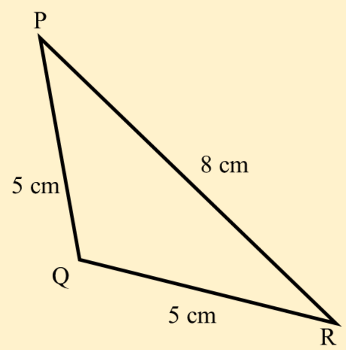
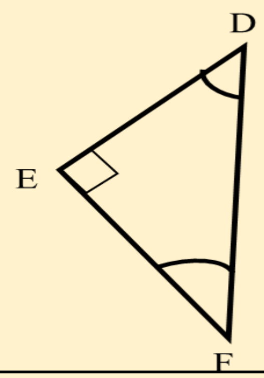
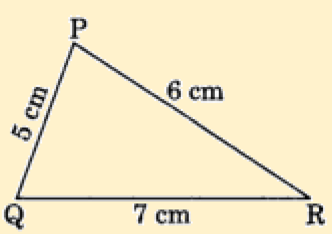
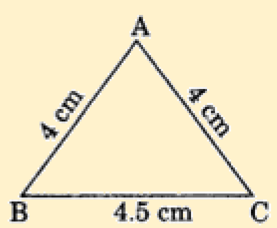
Δ PQR is an acute-angled triangle as all of its angles are acute, i.e. less than 90°.
∠P, ∠Q and ∠R are acute.

Δ MNO is an obtuse-angled triangle as one of its angles is obtuse, i.e., greater than 90° but less than 180°.
∠N is an obtuse angle.

Δ XYZ is an equilateral triangle as all of its angles are equal.
∠X = ∠Y = ∠Z

Quiz: Time

1. In a Δ PQR with base QR, find out the side opposite to ∠Y, the angle made by the sides YZ and XZ, and the vertex opposite to XY.
2. Can an isosceles triangle be an obtuse-angled triangle?
3. State whether the following sentence is true or false.
   In an isosceles triangle, the angles opposite to equal sides are always acute.
4. Classify the following triangles given in Column I to their appropriate type in Columns II and III.

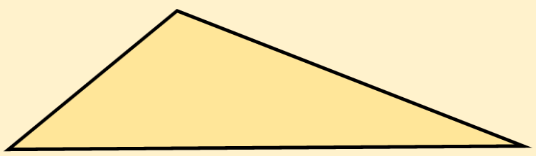
Triangles I	Types	
	On the basis of sides II	On the basis of angles II
		
		
		
		

Median of a Triangle:

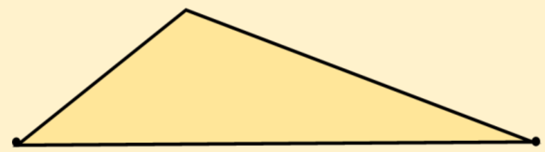
To understand the median of a triangle, let us perform an activity.

Materials Needed: Sheet of paper, Scissors, Ruler, Marker or pencil, Pins or glue

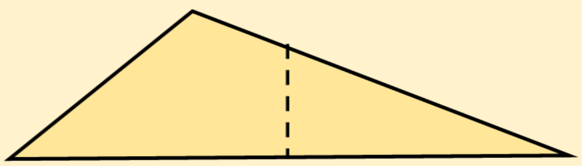
- Take a triangular sheet of thick paper. See figure (i).
- Fold it in such a way that two of its vertices coincide with each other. See the bulleted vertices of figure (ii) that are to be coincided after folding the paper.
- Press the folded paper.
- Observe the crease so formed in the paper. See figure (iii).
- The crease shown by the dotted line is passing through the mid point of the side on which the sheet was folded.
- Fold the paper again along the line passing through the mid point of the side and its opposite vertex.
- Now, observe the crease passing through the vertex and the mid point of its opposite side. See figure (iv).



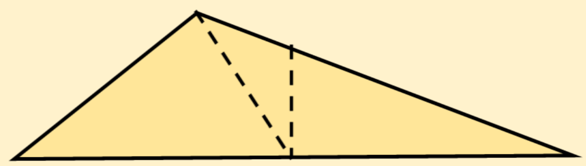
(i)



(ii)



(iii)

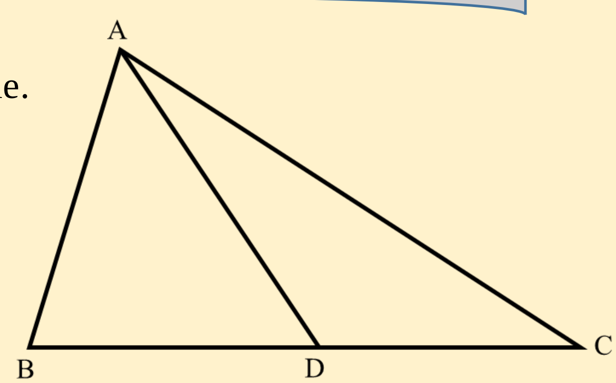


(iv)

The second crease that passes through a vertex and the mid point of the opposite side is the median of the given triangle.
So,

> Median of a triangle is a line segment that joins a vertex of the triangle and the mid point of the side opposite to the vertex.

In the given Δ ABC, line segment AD is the median of the triangle.
D is the mid point of BC.



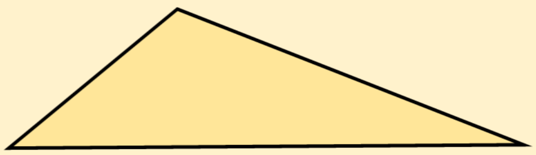
Did you know:
The intersection point of all three medians of a triangle is called centroid of the triangle.

Altitude of a Triangle:

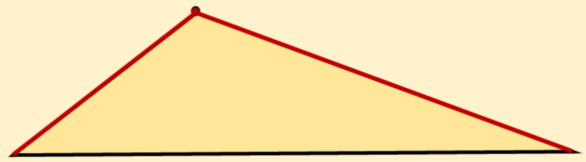
To understand the altitude of a triangle, let us perform an activity.

Materials Needed: Sheet of paper, Scissors, Ruler, Marker or pencil, Pins or glue

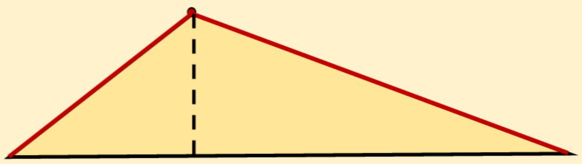
- Take a triangular sheet of thick paper. See figure (i).
- Fold it around one of its vertex in such a way that the two sides meeting at this vertex, coincide with each other. See figure (ii).
- Press the folded paper.
- Observe the crease so formed in the paper. See figure (iii).



(i)



(ii)



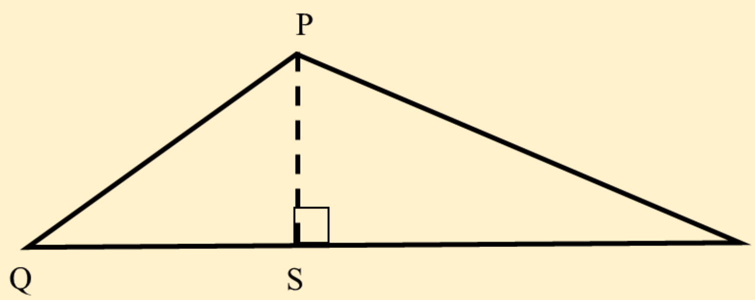
(iii)

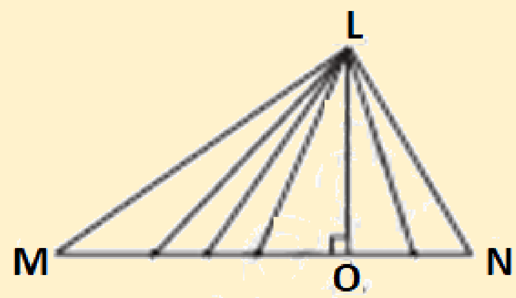
The crease so formed is passing through a vertex, and is perpendicular to the opposite side of the given triangle. This crease is the altitude or the height of the given triangle.

So,

The altitude (or height) of a triangle is a perpendicular segment from a vertex to the opposite side (the base).

In the given Δ PQR, line segment PS is the altitude or height of the triangle. Also, an altitude of a triangle is shortest distance of the base from the vertex opposite to it. In Δ LMN, LO is the shortest distance of MN from the vertex L.



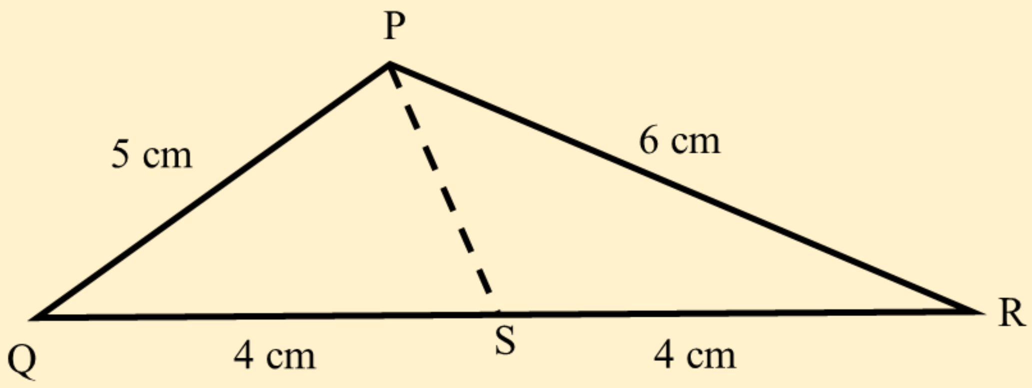


Did you know:

i. The intersection point of all three altitudes of a triangle is called orthocentre of the triangle.
ii. The legs of a right-angled triangle are two of its altitudes.
iii. In an isosceles triangle, the altitude from the vertex opposite to the unequal side bisects the unequal side.
iv. In an equilateral triangle, each altitude bisects the side it intersects.

From Theory to Practice:

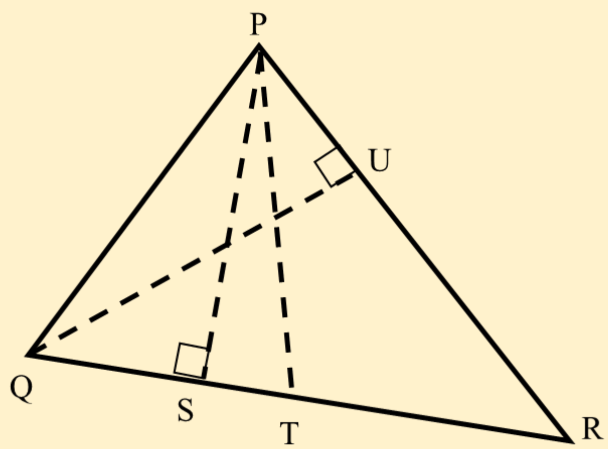
1. In the Δ PQR, QS = SR = 4cm. Then the median in this triangle is
   (a) QS (b) SR (c) PS (d) QR



Solution:

As QS = SR = 4cm, S is the mid point of QR. So, PS is the median in the given triangle.

2. In the triangle PQR, T is the mid point QR. Name the medians and altitudes in this triangle.



Solution:

As T is the mid point of QR, so, PT is the median of this triangle. Also, PS and QU are perpendicular at QR and PR respectively from their respective opposite vertices P and Q. Therefore, PS and QU are the altitudes.

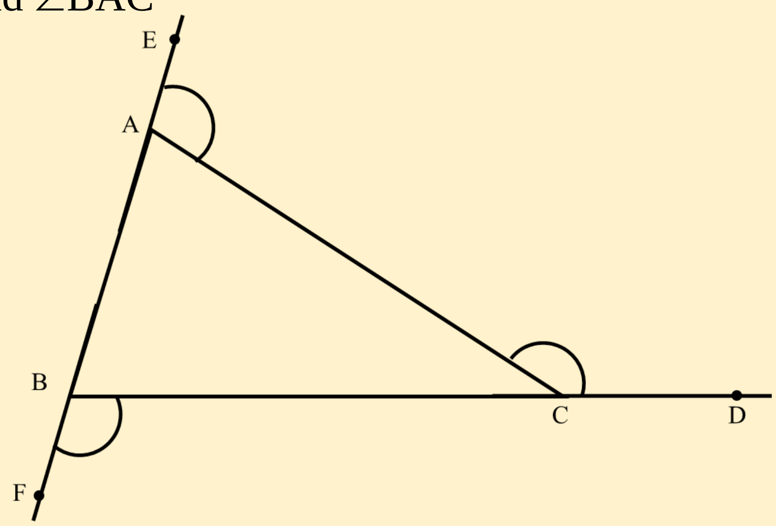
Quiz Time:

i. How many medians can be drawn in a triangle.
ii. Do the medians of a triangle pass through a common point?
iii. Does a median wholly lie inside the triangle?
iv. How many altitudes can be drawn in a triangle.
v. Do the altitudes of a triangle pass through a common point?
vi. Does an altitude wholly lie inside the triangle?

Exterior Angle of a Triangle and its Property:

As the word ‘Exterior’ itself means ‘outside part of something or someone’, an Exterior Angle of a triangle is an angle which is outside the triangle.

In the figure given below, ∠ACD, ∠CAE and ∠CBF are the exterior angles of ΔABC. ∠ABC and ∠CAB are the **interior opposite angles** of the exterior ∠ACD. Similarly, the **interior opposite angles** for ∠CAE and ∠CBF, respectively, are:
∠ABC and ∠CBA
∠BAC and ∠BAC



You might be thinking:

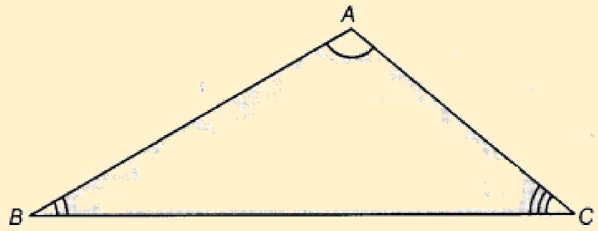
- Whether all the exterior angles of a triangle are of equal measure.
- Do they have any relation with the interior angles of the triangle?

To find out the answer to such questions, let us perform an activity.

Activity:

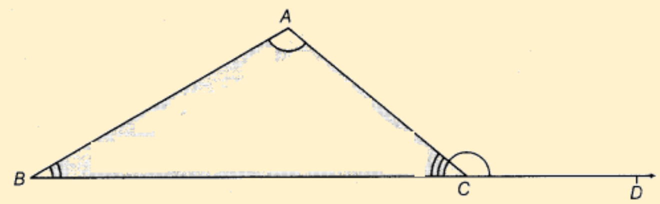
Materials Needed: Cardboard, Scissors, Ruler, Protractor, Marker or pencil, Pins or glue

Steps:



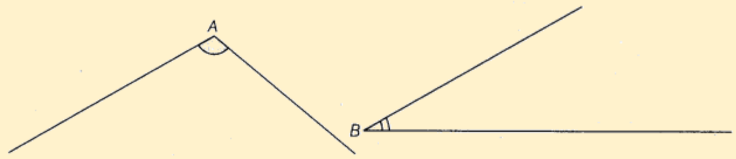
1. Draw a △ABC on the cardboard. Label the vertices A, B, and C.

2. Cut out the △ABC carefully along the lines (sides of the triangle).

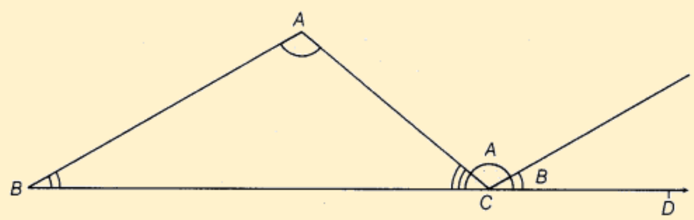


3. Place it on the sheet again. Extend one side of the triangle to form an exterior angle. For example, extend side BC to a point D.

4. Label the exterior angle ∠ACD. This is the angle we will verify.



5. Cut the triangle at the vertices A and B such that cut pieces have angles A and B.

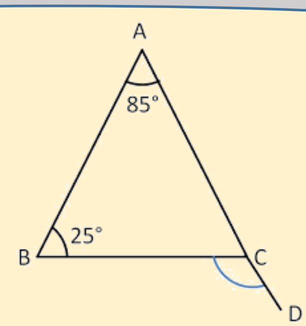


6. Align the cut pieces along the extended side such that ∠CAB and ∠ABC lie adjacent to ∠ACD.

7. You will observe that the sum of ∠CAB and ∠ABC is equal to ∠ACD.
8. Take some more triangles of different measures, and perform this activity. You will find the same property of exterior angles.

So, from this activity, we conclude that:

Exterior angle of a triangle is equal to the sum of its opposite angles.

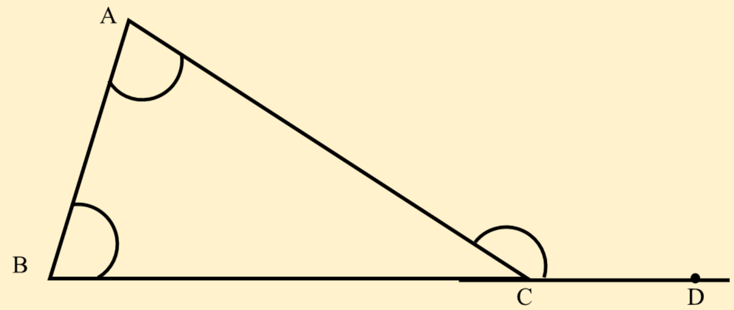


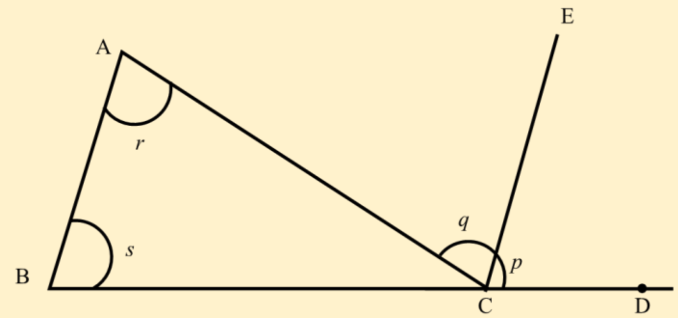
In the given triangle ABC,
the exterior ∠BCD = ∠ABC + ∠CAB = 25° + 85 ° = 110 °

A geometrical argument of just obtained exterior angle property will further prove it:

Geometrical Proof:

Given: △ABC, Exterior ∠ACD





To Prove: m ∠ACD = m ∠CAB + m ∠ABC

Construction: Draw a line CE || AB. Label ∠DCB as *p*, ∠ECA as *q*, ∠CAB as *r* and ∠CBA as *s*.

Proof:

∠*p* = ∠*r*	[CE || AB and AC is a transversal. So, So, alternate interior angles are equal.]
∠*q* = ∠*s*	[CE || AB and BD is a transversal. So, So, corresponding angles are equal.]

On adding both these equations, we get,

∠*p* + ∠*q* = ∠*r* + ∠*s*

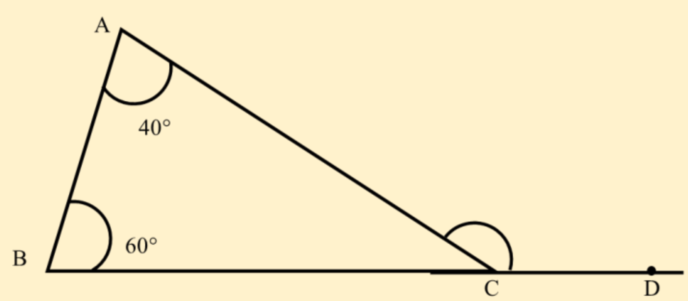
But ∠*p* + ∠*q* = ∠ACD

Therefore, m ∠ACD = m ∠CAB + m ∠ABC

From Theory to Practice:

1. In △ABC, ∠CAB = 40° and ∠ABC = 60°. Find the measure of the exterior angle ∠ACD if BC is extended to D.

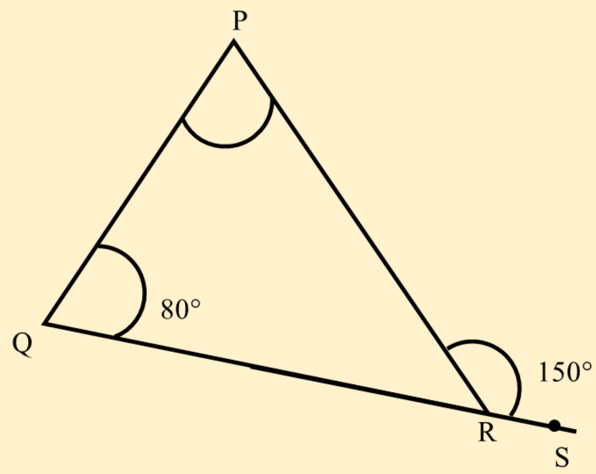
Solution:



By exterior angle property,

∠ACD = ∠CAB + ∠ABC = 40° + 60° = 100°

2. In △PQR, ∠PQR = 80° and the exterior angle ∠PRS is 150°. Find ∠QPR.



Solution:

By the exterior angle property:

∠PRS = ∠PQR + ∠QPR

⇒ ∠QPR = ∠PRS − ∠PQR

On substituting the values of the given angles, we get:

∠QPR = 150° − 80° = 70°

3. In △LMN, ∠LMN = 2∠MLN and the exterior angle ∠LNO is 153°. Find ∠LMN and ∠MLN .

Solution:

Solution:

Let ∠MLN be x°. So, ∠LMN = 2x°

By the exterior angle property:

∠LNO = ∠LMN + ∠MLN

153° = 2x° + x° = 3x°

⇒ x° = 153/3 = 31°

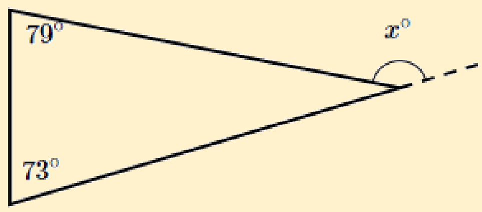
So, ∠MLN = 31° and ∠LMN = 2 × 31° = 62°

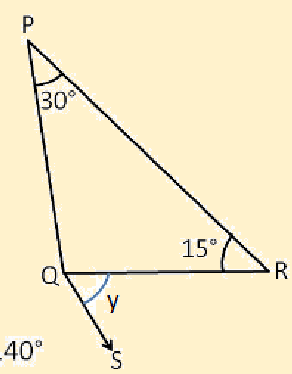
Did you know:

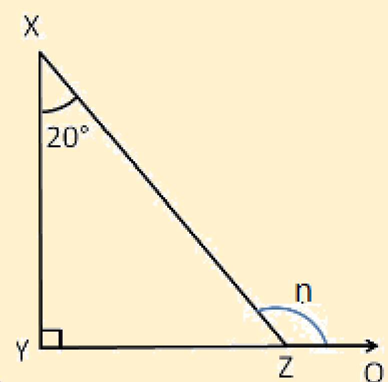
1. The sum of all the three exterior angles of triangle is 360°.
2. In sports like basketball and soccer, the exterior angle theorem helps determine the angle and trajectory of a ball when it bounces off a surface or is kicked or thrown.

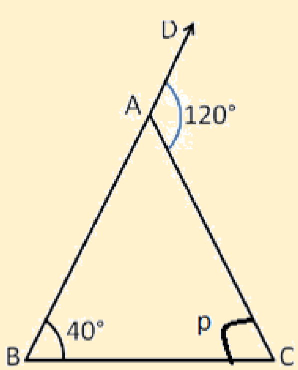
Quiz Time:

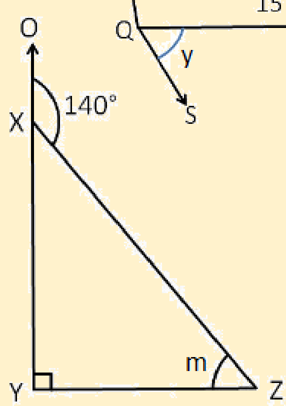
i. Find the unknown angles in the triangles given below.

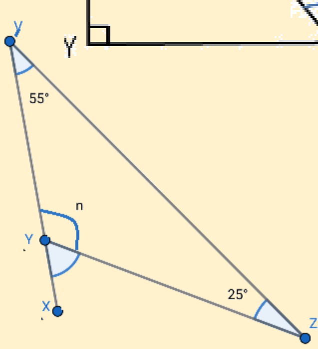












Angle Sum Property:

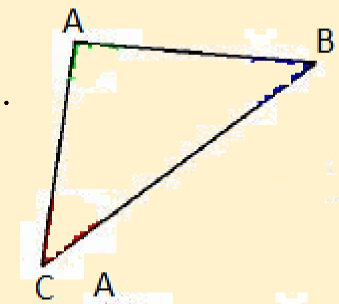
You just studied about exterior angle property in which we use internal angles of the triangle. Have you ever thought about the sum of the internal angles of a triangle? Before we get to know more about it, let us perform an activity.

Activity:

Materials Needed: Cardboard or stiff paper, Scissors, Ruler, Protractor, Pencil or marker, Glue or tape

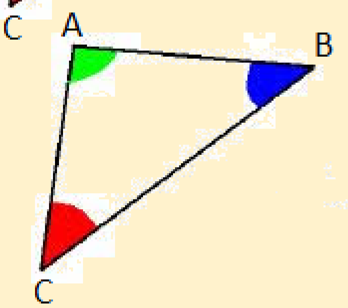
Steps:

1. Draw a triangle on a piece of cardboard. Label the vertices as A, B, and C.

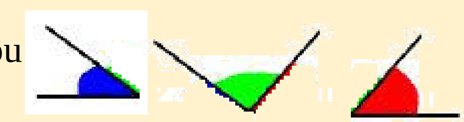


2. Cut out the triangle carefully.

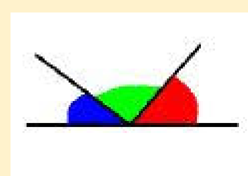
3. Label the angles on the triangle.



4. Cut out each angle of the triangle along lines that extend to the sides. You will end up with three separate pieces, each containing one angle of the triangle.



5. Arrange the three angles so that their vertices touch each other. The three angles now constitute one angle. This angle is a straight angle and so has a measure 180°.



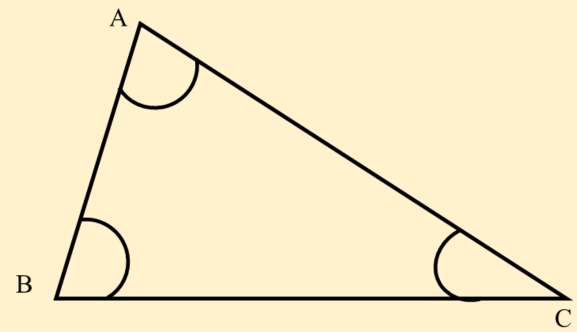
6. Perform this activity for more triangles of different sizes. Check whether you get the same result.

What does this activity demonstrate?

It demonstrates that the sum of all the interior angles of the triangle ABC is 180°.

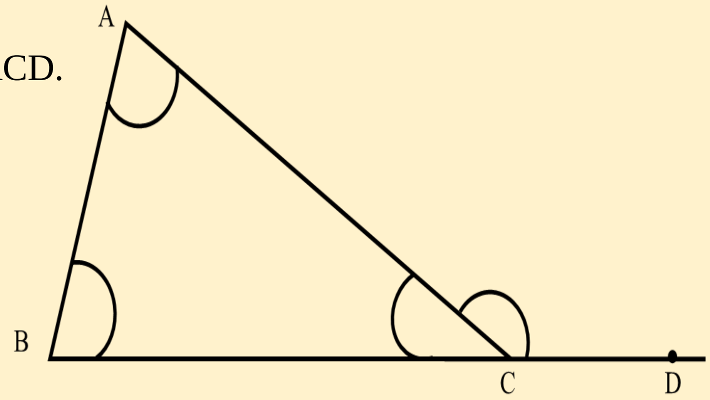
Now, we will verify this property with geometrical arguments.

Geometrical Proof of Angle Sum Property:



Given: △ABC

To Prove: The sum of the interior angles of △ABC is 180°, i.e., ∠A + ∠B + ∠C = 180°.



Construction: Extend BC to D to make an exterior angle ∠ACD.

Proof:
∠ACD = ∠ABC + ∠CAB …(1) [Exterior angle Property]

∠ACB + ∠ACD = 180° [Linear pair of angles]
⇒ ∠ACD = 180° – ∠ACB

Substituting this value of ∠ACD in equation (1), we get

180° – ∠ACB = ∠ABC + ∠CAB

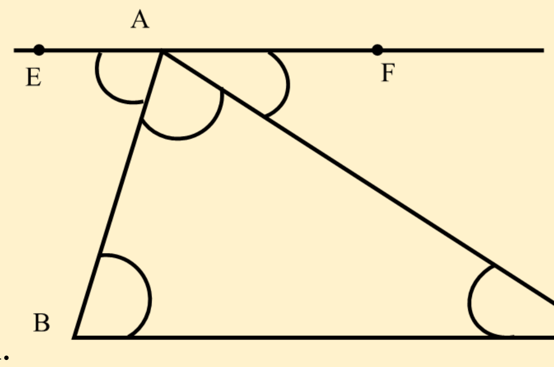
⇒ 180° = ∠ABC + ∠CAB + ∠ACB

⇒ ∠ABC + ∠CAB + ∠ACB = 180°
Hence, proved.
So, we can state that:

The sum of all the interior angles of the triangle ABC is 180°.

Do you know there is another proof of angle sum property? Let us discuss this.

Alternate Proof:



Construction: Draw a line EAF parallel to the side BC of the triangle ABC.

Proof :
∠EAB = ∠ABC and ∠FAC = ∠ACB …(1)
[Alternate interior angle are equal.]
∠EAB + ∠BAC + ∠FAC = 180° …(2)
[Sum of angles made on a straight line is 180°.]
From (1) and (2), ∠ABC + ∠CAB + ∠ACB = 180°. Hence, proved.

From Theory to Practice:

1. In △ABC, ∠A = 45° and ∠B = 65°. Find ∠C.

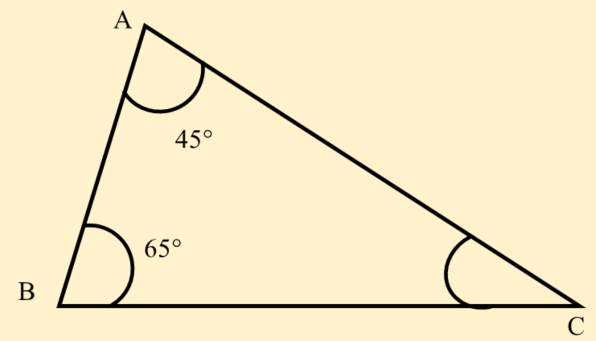
Solution:

By angle sum property, we know that:

∠A + ∠B + ∠C = 180°

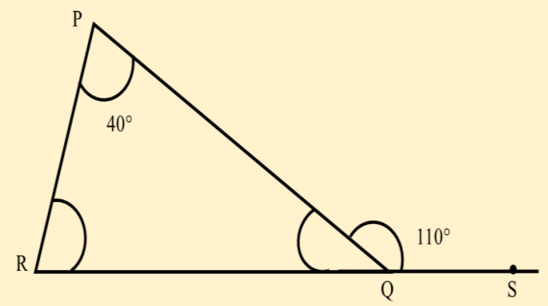
But we have ∠A = 45° and ∠B = 65°.

So, 45° + 65° + ∠C = 180° ⇒ ∠C = 180° – 105° = 75°



2. In △PQR, the exterior angle at Q is 110 ° and ∠P = 40° . Find ∠R.

Solution:



The exterior angle at Q is equal to the sum of the two non-adjacent interior angles:

∠P + ∠R = 110°

Substitute the given value of ∠P:

40° + ∠R = 110°

Solve for ∠R: ∠R = 110° − 40° = 70°

3. In △ABC, ∠A = ∠B and ∠C=80°. Find ∠A and ∠B.

Solution:

Since ∠A =∠B, let's denote ∠A = ∠B = x°.

Using the angle sum property:∠A + ∠B + ∠C = 180°

Substitute the given and equal angles:

x + x + 80° = 180°

⇒ 2x + 80° =180°

⇒ 2x = 180° − 80° = 100°

⇒ x = 100°/2 = 50°

Quiz Time:

i. Can a triangle have 45°, 105° and 55° as its internal angles?

ii. Mohan wants to find the measure of the third angle of a triangle ABC in which ∠ABC = 55° and ∠ACB = 75°. Can you help him?

iii. If the angles of a triangle are in the ratio 2:3:4, determine the values of these angles.

Two Special Triangles : Equilateral and Isosceles:

While discussing types of triangles at the beginning of this lesson, we identified two special types: Equilateral and Isosceles Triangles. They have unique properties that make them special.
Let us discuss them one by one with the help of activities.

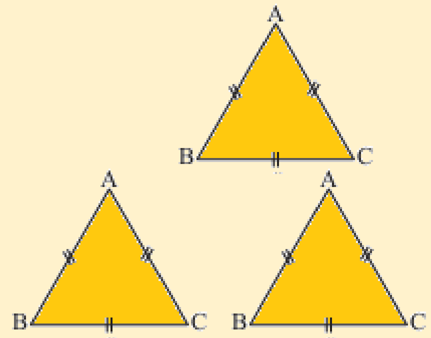
Equilateral Triangle:

Activity 1:

Materials Needed: Cardboard or stiff paper, Scissors, Ruler, Protractor, Pencil or marker, Glue or tape

Steps:

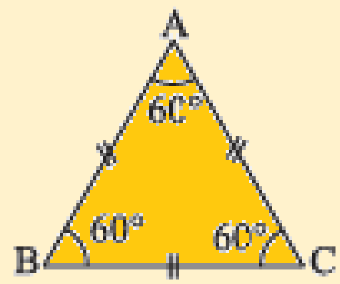
1. Draw an equilateral △ABC on a piece of cardboard.



2. Cut out two copies of the triangle carefully.
3. Keep one of them fixed. Place the second triangle on it. It fits exactly into the first.
4. Now, place the second triangle on the first one in such a way that vertex B of second triangle coincides with the vertex A of first triangle.
5. You will observe that that still the second triangle exactly fits into the first.
6. Turn it round in any way and still they fit with one another exactly.
7. It shows that angles of an equilateral triangle have equal measure.

In an equilateral triangle,
(i) all sides have same length.
(ii) all angles have same measure.

Let the measure of an angle of an equilateral triangle be x.
So, all three angles will have same measure as x.
By angle sum property,
x + x + x = 180°
⇒ 3x = 180°
⇒ x = 60°



Therefore, in an equilateral triangle, each angle measures 60°.

Did you know:
In an equilateral triangle, each altitude bisects the side it intersects.

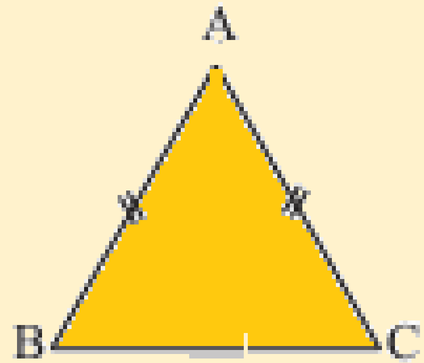
Isosceles Triangle: We know that a triangle in which two sides are of equal length is called an isosceles triangle.

Activity 2:

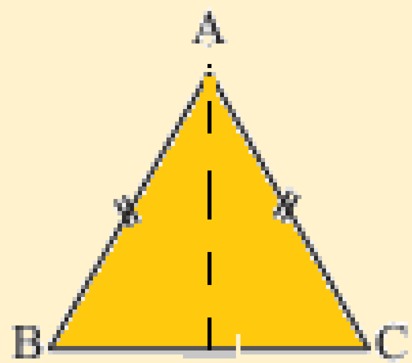
Materials Needed: Cardboard or stiff paper, Scissors, Ruler, Protractor, Pencil or marker, Glue or tape

Steps:

1. Draw an isosceles △ABC in which AB = AC on a piece of cardboard.



2. Fold it in such a way that vertex C coincides with vertex B. Press the folded triangle.



3. What do you observe? You can observe that ∠C completely coincides with ∠B, and the crease so formed in the folded triangle divides it into two equal parts.

4. It shows that ∠B = ∠C.

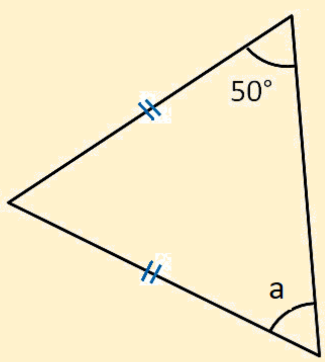
So,

In an isosceles triangle,
(i) two sides have same length.
(ii) base angles opposite to the equal sides are equal.

Did you know:
In an isosceles triangle, the altitude from the vertex opposite the unequal side bisects the unequal side.

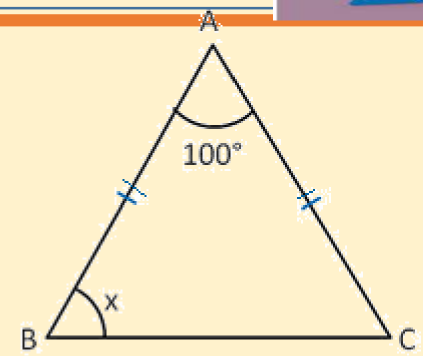
From Theory to Practice:

1. Find the angle ‘a’ in the following triangle.



Solution:
Since the given triangle is isosceles, their opposite angles will be equal. Therefore, angle ‘a’ is 50°.

2. Find the angle ‘x’ in the adjoining triangle.



Solution:

Since the given triangle is isosceles, their opposite angles will be equal. Therefore, ∠C = x. Now, by angle sum property,

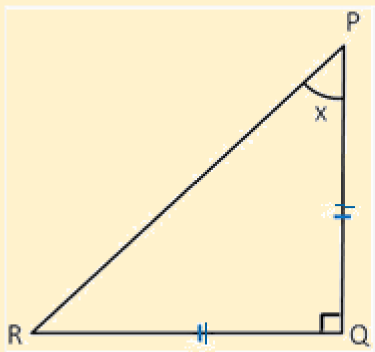
∠A + ∠B + ∠C = 180°

⇒ 100° + x + x = 180°

⇒ 2x = 80°

⇒ x = 40°

3. Find the angle ‘x’ in the adjoining triangle.



Solution:

Since the given triangle is isosceles, their opposite angles will be equal. Therefore, ∠R = x. Now, by angle sum property,

∠P + ∠Q + ∠R = 180°

⇒ x + 90° + x = 180° [It is given that ∠Q is a right angle. So, ∠Q = 90°]

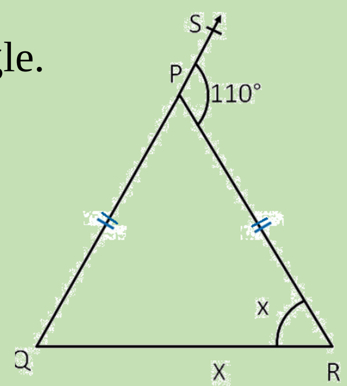
⇒ 2x = 90°

⇒ x = 45°

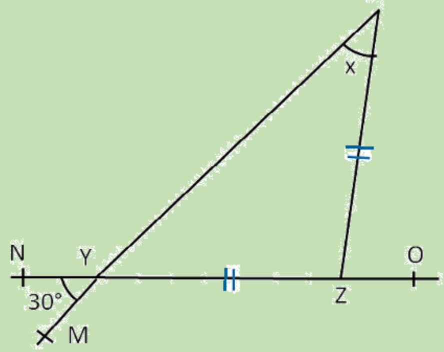
Quiz Time:

i. State whether the following statement is true or false.
An equilateral triangle is a special case of an isosceles triangle.

ii. Find the value of angle ‘x’ in the adjoining figure.



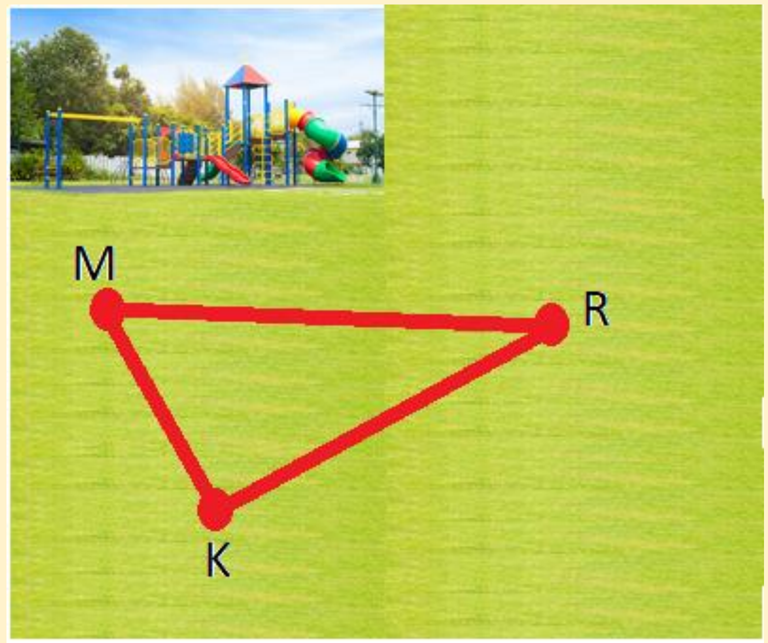
i. Find the value of angle ‘x’ in the adjoining figure.



Sum of the Lengths of two Sides of a Triangle:

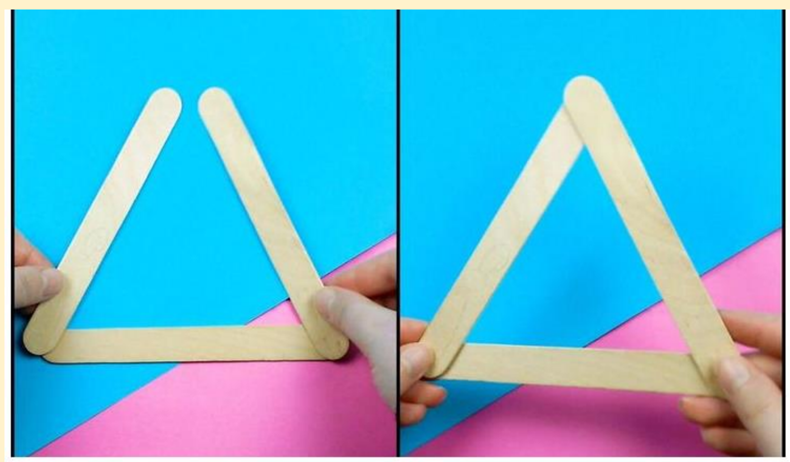
So far, we have discussed different triangles and properties primarily related to their angles, like exterior angle property, angle sum property, etc. But have you ever thought about how a triangle is intrinsically related to the lengths of its sides? Let us see how.

Krishna, along with his friends Meera and Rahul, is playing on the ground. They are standing at three different positions KK, MM, and RR, respectively, as shown in the diagram. Krishna wants to reach Rahul as quickly as possible. He decides to go via Meera. Was this the best route? No, it wasn't. He would have reached Rahul faster if he had gone directly.
So, what does this activity demonstrate?
Observe that the positions of Krishna and his friends forms a triangle.
In **Δ KMR, KM + MR > KR**



You can further explore this relationship through a 'Stick Activity'.

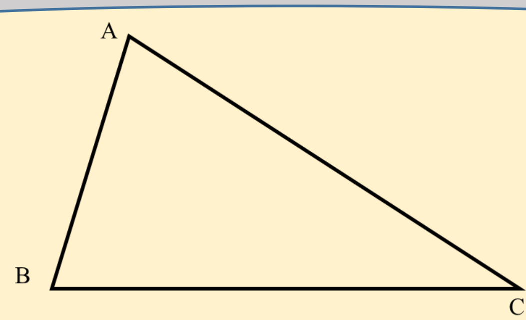
- Take some small sticks of length say 5 cm, 7 cm, 10 cm, 12 cm, 15 cm, etc.
- With different combinations of these sticks, try to form triangles.
- You will find that a triangle can be formed with any three sticks only if the sum of the lengths of any two sticks is greater than the length of the third stick.



You can verify this result by drawing different triangles in your notebook, and then measuring their sides too.
So, we can conclude that:

The sum of the lengths of any two sides of a triangle is greater than the third side.

In Δ ABC,
AB + BC > AC
AB + AC > BC
BC + AC > AB



From Theory to Practice:

1. Given three sides a = 3 cm, b = 4 cm, and c = 8 cm, determine if they can form the sides of a triangle.

Solution:

Given three sides a = 3 cm, b = 4 cm, and c = 8 cm.
$a + b = 3 + 4 = 7 < c$ (8 cm)
So, a triangle cannot be formed with these sides.

2. Given three sides a = 4 cm, b = 8 cm, and c = 9 cm, determine if they can form the sides of a triangle.

Solution:

Given three sides a = 4 cm, b = 8 cm, and c = 9 cm.
$a + b = 4 + 8 = 12 > c$ (9 cm)
$a + c = 4 + 9 = 13 > b$ (8 cm)
$b + c = 8 + 9 = 17 > a$ (4 cm)
So, a triangle with these sides can be formed .

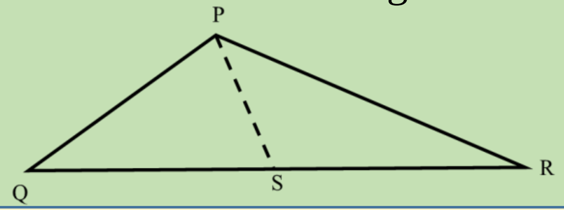
3. Given two sides of a triangle as 5 cm and 12 cm, what is the smallest possible integer length for the third side?

Solution:

Let the given side be a = 5 cm and b = 12 cm, and the third side be c
Given three sides a = 5 cm, b = 12 cm, and the third side = c cm.
$a + b = 5 + 12 = 17 > c$
$a + c = 5 + c > b = 12$
$b + c = 12 + c > a = 4$
From these relations, we have $c < 17$, $5 + c > 12$ and $12 + c > 4$
That means,
$c < 17$
$c > 7$
$c > -8$, which is not possible as length cannot be negative.
So, $7 < c < 17$
Therefore, the smallest integral value of c is 8 cm.

Quiz Time:

i. Can we say that a triangle can be formed if the sum of any two of its angles is greater than the third angle?
ii. AM is a median of a triangle ABC. Is AB + BC + CA > 2 AM?



Right-angled Triangles and Pythagoras Property:

We just discussed the relationship between the lengths of a triangle's sides. Special triangles, such as equilateral and isosceles triangles, have unique relationships among their sides. In equilateral triangles, all sides are equal in length. In isosceles triangles, two sides are equal.

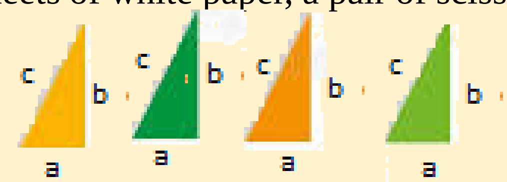
Do you know that for a right-angled triangle too, there is a unique relation between its sides?
To discuss this relation, let us perform an activity.

Activity 1:

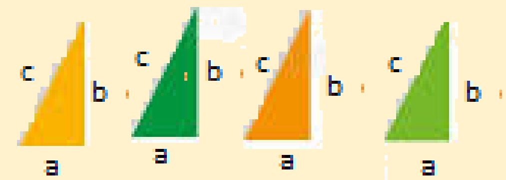
Materials Needed: Four pieces of different coloured cardboard, two sheets of white paper, a pair of scissors, a geometry box, a tube of glue

Steps:

1\. Create eight identical copies of a right-angled triangle, choosing any size you prefer.

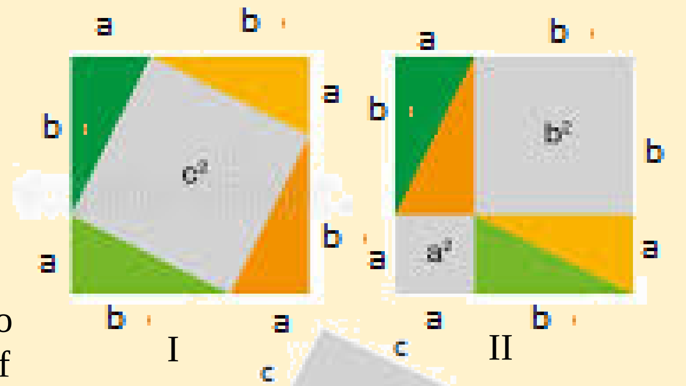


1\. Label their sides as hypotenuse of ‘c’ units and legs measuring ‘a’ units and ‘b’ units.

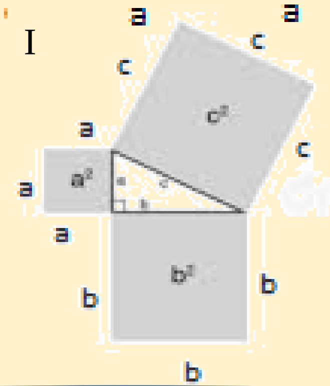


1\. On a sheet of paper, draw two identical squares, each with sides of length a + b.

2\. Arrange four triangles in one square and the remaining four triangles in the other square, as shown in the adjoining diagram.



The squares are identical, and the eight triangles inserted are also identical. So, the uncovered area of square I = Uncovered area of square II.
That means, the area of inner square of square I = The total area of two uncovered squares in square II.

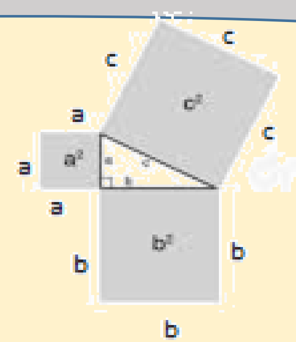


Or $c^2 = a^2 + b^2$ This is Pythagoras property. So, we can state it as:

In a right-angled triangle, the square on the hypotenuse = sum of the squares on the legs.

Activity 2:

You can further verify this property by drawing a right-angled triangle, and then drawing squares on all its three sides. You will find the same result. **The area of the square made on the hypotenuse will be equal to the sum of the areas of the squares made on the legs of the right-angled triangle.**



Did you know:

1. Pythagoras of Samos (c. 570–495 BCE) was an ancient Greek philosopher and mathematician. He was a pioneering figure in ancient Greek mathematics and philosophy. His theorem, which describes the relationship between the sides of a right-angled triangle, is a fundamental concept in geometry and has enduring significance in both academic and practical contexts, and has been named after him.
2. Even the converse of Pythagoras Property is true. That means if in a triangle, square of one side is equal to the sum of the squares of other two sides, then the triangle is a right-angled triangle.
3. A Pythagorean triplet consists of three positive integers a, b, and c such that they satisfy the Pythagoras Property. For example, (3, 4, 5) is a Pythagorean triplet as $3^2 + 4^2 = 5^2$

From Theory to Practice:

1. Determine whether the triangle whose lengths of sides are 6 cm, 8 cm, 10 cm is a right-angled triangle.

Solution:

$10^2 = 10 \times 10 = 100$, $8^2 = 8 \times 8 = 64$, $6^2 = 6 \times 6 = 36$

We can observe that $10^2 = 8^2 + 6^2 = 64 + 36 = 100$. Hence, the triangle formed with the given sides is a right-angled triangle.

2. A right-angled triangle has legs of lengths 3 units and 4 units. Find the length of the hypotenuse.

Solution:

By Pythagoras Property, in a right-angled triangle,

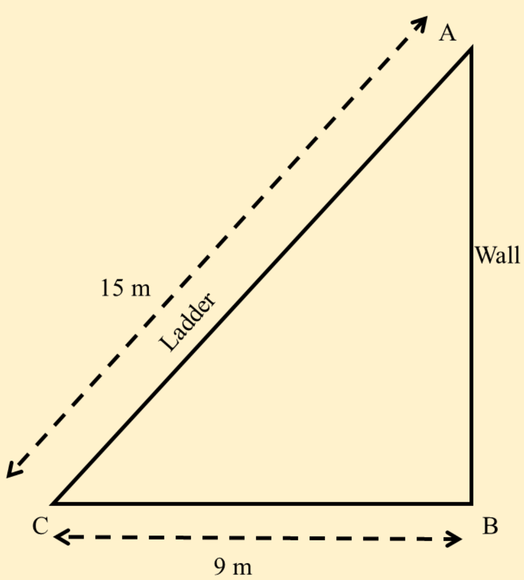
$(\text{Hypotenuse})^2$ = Sum of the squares of Legs

$(\text{Hypotenuse})^2 = 3^2 + 4^2 = 9 + 16 = 25$

$(\text{Hypotenuse})^2 = 5^2$

Hypotenuse = 5 units

3. A ladder 15 units long leans against a wall. If the bottom of the ladder is 9 units away from the wall, how high up the wall does the ladder reach?



Solution:

When a ladder leans against a wall, a right-angled triangle is formed as shown in the adjoining diagram.

By Pythagoras Property, In right-angled triangle ABC,

$AC^2 = AB^2 + BC^2$

$15^2 = AB^2 + 9^2$

$225 = AB^2 + 81$

$AB^2 = 225 - 81 = 144$

$AB^2 = 12^2$

AB = 12 units

Therefore, height of the wall is 12 units.

Quiz Time:

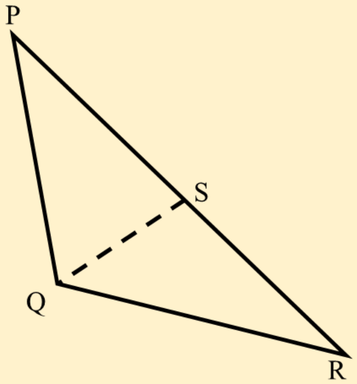
i. A ramp is to be built to cover a horizontal distance of 20 meters and an elevation of 15 meters. Find the length of the ramp.

ii. Can the sum of squares of two natural numbers be equal to the square of another natural number?

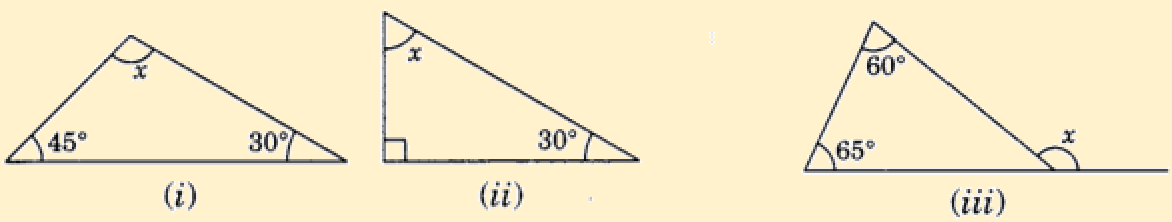
iii. The length of the diagonal of a square is 6 cm. Find the sides of the square.

Chapter Exercise:

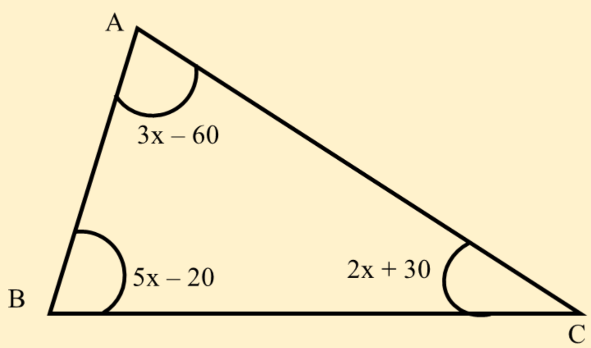
1. In ΔXYZ, write the following:
   (a) Angle opposite to side YZ.
   (b) The side opposite to ∠XYZ.
   (c) Vertex opposite to side XZ
2. Can you have a triangle with all the angles less than 60°? Explain your answer.
3. In the given figure, name the median and the altitude. Here S is the midpoint of PR



4. In the given diagrams, find the value of x in each case.



5. Which of the following cannot be the sides of a triangle?
   (i) 2.3 cm, 4.5 cm, 5.6 cm
   (ii) 3.4 cm, 4.6 cm, 10 cm
   (iii) 3.8 cm, 5.9 cm, 2 cm
6. In the given figure, find x.



Chapter Exercise:

7. One of the equal angles of an isosceles triangle is 45°. Find all the angles of this triangle.
8. Two sides of a triangle are 5 cm and 8 cm. What can be the maximum integral length of its third side to make the triangle possible?
9. PS is the median of a ΔPQR, prove that $PQ + QR + PR > 2PS$.
10. The sides of a triangle are in the ratio 3 : 4 : 5. State whether the triangle is right-angled or not.